Directional Multiresolutional Image Analysis

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Efficient representations of signals require that coefficients of functions that represent the regions of interest are sparse. This also means that we can reconstruct the original function with a smaller set of basis functions with better accuracy.

Wavelets have had great success in signal processing because of their good approximation properties [1] and ability to pick up discontinuities efficiently in one dimensional piecewise smooth functions, the discontinuities here are zero-dimensional or point discontinuities. In two dimensional functions, however, discontinuities are one-dimensional, like discontinuities occurring over edges or curves. Intuitively, wavelets in 2-D obtained by a tensor product of one-dimensional wavelets will be good at isolating the discontinuity at an edge point, but will not see the smoothness along a contour. For example in Figure 1 the wavelet transform of the vertical edge is represented by coefficients in only one directional orientation, whereas the diagonal edge is represented by significant coefficients in all the directional orientations.

Numerous methods have been proposed independently to overcome the problem. A major part of the work done this summer was to study and comprehend the rapidly growing literature on directional multiresolutional image analysis and classify the different methods based on some common approach they use. The other part of the work was to study the different properties of these methods and choose appropriate transforms for either image compression or denoising.

Classification of Techniques

We have classified the different directional multiresolutional image analysis techniques as (1) Adaptive, (2) Radon-based (3) Filter bank based.

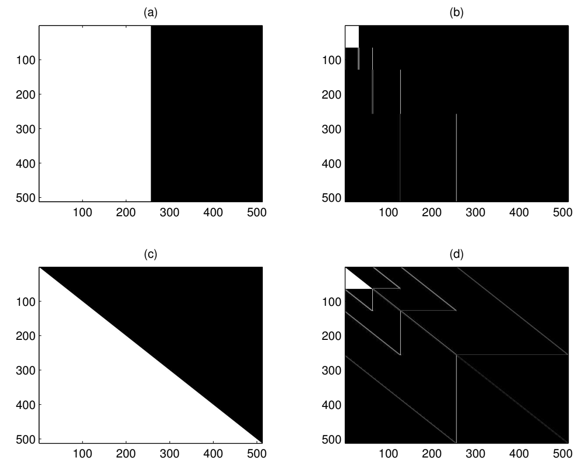

Figure 1: (a) a vertical edge image, (c) a diagonal edge image, (b) and (d) are their wavelet transforms respectively.

Adaptive Techniques

Adaptive techniques are techniques where the directional component of an image is adaptively estimated and the transform is steered based on the estimate. For example the bandlet transform of Pennec and Mallat [2] links the significant wavelet coefficients along a discontinuity and represents it as a smooth 1-D curve.

Radon-based Techniques

We can classify the curvelet and ridgelet transforms as Radon based transforms because both these transforms use the Radon transform for directionality. The ridgelet transform was developed by Candes and Donoho [3] to overcome the disadvantage of the 2-D wavelet transform. The ridgelet transform uses the radon transform to map a line singularity to a point singularity and uses the wavelet transform to deal with the point singularity effectively. The continuous ridgelet transform (CRT) in $\mathbf{R}^2$ space is defined as

$$CRT_f(a,b,\theta) = \int_{\mathbf{R}^2} \psi_{a,b,\theta}(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} \qquad (1)$$

where the ridgelets are wavelets along a directional component as

$$\psi_{a,b,\theta}(\mathbf{x}) = a^{-1/2}\psi((x_1\cos(\theta) + x_2\sin(\theta) - b)/a) \quad (2)$$

Ridgelet transforms are very effective in representing objects with singularities along lines.

For practical applications, however, we require discrete implementations of the ridgelet transform that leads to algorithmic implementations. This is a challenging problem. Since the radon transform is polar in nature, we cannot implement direct discretizations of continuous formulae. The discretization implemented in this work was developed by Do [4].

The curvelet transform was developed by Candes and Donoho [5]. It was shown to achieve optimal approximation behavior in a certain sense for 2-D piecewise smooth functions in $\mathbf{R}^2$ where the discontinuity is along a curve in the C^2 space. The attractive part of the curvelet transform is that it has good non-linear convergence properties.

The curvelet transform of an image is obtained by the following steps: (1) Subband decomposition of the image into a sequence of subbands by using a pyramidal tree structured filter-bank. (2) Windowing each subband into blocks of appropriate size, depending on the center frequency of the subband. (3) Applying the discrete ridgelet transform on these blocks. The idea behind windowing the subbands is that windowed parts of smooth lines look straight, and these straight parts can be analyzed by a ridgelet transfrom.

Filter Bank Techniques

The discrete algorithmic implementation of the curvelet transform poses many problems. Since it uses windowing of the subband coefficients it may lead to blocking effects, and if we use overlapping windows the redundancy of the transform increases. The other problem with the curvelet transform is that it uses the ridgelet transfrom and as mentioned before the ridgelet transform cannot be efficiently implemented for discrete images.

To overcome this problem Do and Vetterli proposed the pyramidal directional filterbank (PDFB) also known as the contourlet transform [6]. This approach overcomes the block based approach of the curvelet by using a directional filter bank [7]. The contourlet transform first applies a multiscale decomposition on the image and then the local radon decomposition is obtained by a directional filter bank.

Results

We performed denoising by hard thresholding of the transform coefficients. The contourlet trasform performed better than the wavelet transform. For the standard images Lena, Barbara and Mandrill we obtained a perfromance gain of 2.1 dB, 0.7 dB and 1.8 dB respectively.

In the future we would like to be able to understand and present the different directional and multiresolutional transforms in a way such that their differences and similarities are made explicit. This would be useful for selecting transforms for specific image processing tasks.

Currently we are working on using the contourlet and curvelet transforms for image denoising. We hope to obtain better results by using the directional properties of these transforms. We are also working on using these transforms for compression instead of the 2-D wavelet transforms.

Acknowledgements

Los Alamos Report LA-UR-04-xxxx.

References

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